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gives for the

1st order	$-0''\cdot0096363$
3rd „	$-0\cdot0012981$
5th „	$-0\cdot0001487$

from which we may estimate that the terms of higher orders will not exceed $-0''\cdot00002$.

The equation

$$\left[\frac{da}{dt}\right]_{00}=0$$

gives

$$\sin\phi\tfrac{1}{2}A_1^{(s)}+\cos\phi B_0^{(c)}=0,$$

Substituting the respective numbers I get

$$-0\cdot000447016+0\cdot000447108=0,$$

error—

$$+0\cdot000000092.$$

In making a similar operation on the figures, Professor Hill has given in his example of *Mercury*, perturbated by *Venus*, the difference is more considerable—viz.,

$$-0\cdot00075,$$

probably through a misprint of a figure in the value of R_0 or S_0.

In some parts of the calculations I have used eight decimals, and S and S_1 refer to the original numbers.

Sydney: 1891 *November* 1.

On the Spectrographic Method of determining the Velocity of Stars in the Line of Sight. By H. C. Vogel, Foreign Associate.

The experiments made at Potsdam in 1887 showed that, as a result of the extremely sensitive photographic methods employed, a sufficiently great dispersion could be made use of to readily detect and measure the displacement of the spectral lines produced by the motion of the stars in the line of sight. It very soon became clear that the measurement of the stellar spectra admitted of a far greater exactness than the direct observations, and that the disturbances of the atmosphere—the chief cause of the difficulties of the direct method—exert their influence in a lesser degree on the photograph. The very numerous measurements on more than two hundred negatives of forty-seven stars,

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which are now available, have confirmed this result, and show further that the exactness of the measurements far surpasses the expectations based on the first plates taken with a provisional apparatus, and that the definitive observations have reached a degree of accuracy which in some cases is surprising.

This great accuracy has been secured by an advantageous construction of the apparatus, by its very exact adjustment, and especially by the peculiar methods adopted in measuring the photographs. I have already published several communications on this spectrographic method, viz. one in *Ast. Nach.*, No. 2896, a further one announcing the discovery of the motion of *α Virginis* (*Ast. Nach.*, No. 2995), and more recently an article on the employment of iron as a comparison spectrum in spectrographic researches (*Sitzungsberichte der Akademie zu Berlin*, June 4, 1891).

The reduction of the observations and measurements are at present nearly completed, but the passing through the press will still require several months, and therefore I now take the liberty of presenting a short review of the investigation and its chief results, in the hope that this summary, and more especially the explanation of the method of measuring the plates, may be of value and interest to many who are employed in similar lines of work.

In the construction of the apparatus the following points were taken into consideration: great stability for the smallest possible weight; suitable dimensions of prisms, collimator, and camera objectives, in order to preserve sufficient brightness with the greatest possible dispersion; accurate adjustment of the photographic plate in the focal plane of the camera objective; and exact keeping of the star on the slit of the spectrograph. I have endeavoured to satisfy the first two conditions by having the frame made of cast steel, and by giving it a form which offered the greatest possible resistance to flexure. The most suitable dimensions for collimator and camera objective, for the 12-inch refractor to which the spectrograph was to be applied, were found to be 408 mm. focal length for 34 mm. aperture.

The two Rutherfurd compound prisms have the following dimensions: Height (length of the refracting edge) 35 mm., breadth (perpendicular distance from the refracting edge of the flint-glass prism) 45 mm. They are of the most colourless glass obtainable, and the dispersion from F to H amounts for each to about 5°. The camera is constructed of sheet steel; the plate-holders are of brass and can be rigidly connected with the camera. The adjustment of the photographic film in the focal plane of the camera objective is accomplished by a motion of the latter, and can be effected with an accuracy of a fraction of a millimetre. In order to facilitate keeping the image of the star exactly on the slit, a small telescope is so connected with the apparatus that it receives the light which is reflected from the front side of the first prism.

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The slit, illuminated by the Geissler tube, which furnishes the comparison spectrum, appears in this telescope as a narrow line of light with the star in the middle, and may be readily so held by means of the slow motions of the refractor. A cylindrical lens is not employed, since, with the slit set parallel to the line of the diurnal motion, a slight widening of the linear stellar spectrum can be readily effected by changing the rate of the driving clock. A very important point in the adjustment of this apparatus is that the optical axis of the collimator shall fall in the prolongation of that of the refractor; this is easily and accurately effected by three screws.

By means of a table giving the proper setting for various degrees of temperature the plane of the slit is, before each observation, adjusted to lie in the focus for the rays of the wave-length of Hγ, for which the prisms are set at the angle of minimum deviation, and which appear on the middle of the negative. It is of great importance that the latter be precisely adjusted in the focus of the camera objective. This is likewise effected before each observation according to a table with the argument of the thermometer attached to the instrument. This table has been determined by many experiments, as well as by artificial warming of the apparatus. A false adjustment, which exerts a great influence on the sharpness of the images, and hence on the subsequent measurements, can be at once detected by the lack of distinctness of the image of the artificial hydrogen line, and, similarly, changes of the apparatus which occur during the exposure reveal themselves in the altered appearance of the comparison line.

In nearly all the observations hydrogen has been chosen for the comparison spectrum. The Geissler tube was placed directly in the cone of rays of the refractor, at a distance of 40 cm. from the slit, and was set at right angles to the optical axis of the refractor as well as to the slit, and therefore its light is to be regarded as dispersed on reaching the slit. The tubes used were very thin, so that only a comparatively small amount of light (17 per cent.) was lost in passing through them.

A further advantage of this arrangement of the Geissler tube lies in the fact that an exact adjustment of its position is not necessary, and any slight changes in its place during the exposure, due to an altered position of the instrument, can exert no injurious influence; for one can readily see that with an adjustment even several degrees false, the slit would still appear fully illuminated.

At first the time of exposure was regulated according to the brightness of the star and to the chosen width of slit, but later it appeared advantageous to leave the width unchanged at 0·02 mm. and to give a uniform exposure of one hour. The observer then, however, varied the width of the spectrum to correspond with the brightness of the star. The performance of the apparatus has been tested on the Sun by direct observa-

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tions and by photographs. In the vicinity of Hγ, where the spectrum is sharpest, nearly all the lines are visible which are contained in Rowland's large photograph of the solar spectrum, in spite of the comparatively small dimensions of the apparatus. Of course very close double lines cannot be separated if the width of the slit be greater than that corresponding to the distance between the lines.

One perceives less detail on the photograph than by direct observation, since, on account of the extraordinary delicacy of the lines, some details are lost through the coarseness of the silver grains. The excellent prisms would admit of a much greater magnification of the spectrum by the use of a camera objective of longer focus, and thus make possible the production of still sharper photographs, if the light-power of the refractor were not too small and the observations therefore necessarily limited to the brightest stars. I have given in *Ast. Nach.*, No. 2896, a collection of about fifty lines on one of the solar negatives which lie between 431·4 and 436·8 *μμ*, and may remark further that my assistant, Dr. Scheiner, has been able to measure 290 lines in the spectrum of *α Aurigæ* between 412·4 and 466·8 *μμ*.

In order to decide further as to the correct adjustment of the apparatus, numerous photographs of the Moon's spectrum, with that of hydrogen for comparison, have been made, and show an absolute coincidence of the corresponding lines. Two photographs of *Venus* are not without interest as showing the accuracy of the measures as well as the correctness of the adjustment:

	Observed Velocity.	Calculated Velocity.	
1889 Jan. 2	−7·8	−7·4	English miles.
Feb. 10	−7·4	−7·8	

The measurement of the spectra is accomplished with the aid of a microscope under the employment of magnifying powers from 7 to 35.

The table of the microscope carries a sliding apparatus, movable by a fine micrometer screw of 0·25 mm. pitch, to which the negative is firmly clamped. The periodic and progressive errors of this screw have been accurately determined. By means of a large number of measurements in the solar spectrum it has been found that one revolution of the screw corresponds to a difference in wave-length of 0·324 *μμ*. With this value can now be computed g, the value in miles per second of velocity of one revolution of the screw, from the formula $g = \frac{\Delta\lambda}{\lambda} V$, where V is the velocity of light and λ the wave-length of Hγ, 434·07 *μμ*. The result is $g = 139{\cdot}13$ miles. This value, strictly speaking, holds good only for the temperature at which this standard photograph

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of the solar spectrum was taken, since the dispersion varies with the temperature. A small correction has, therefore, been applied to all the photographs made at a different temperature. For stars of the second type, however, a method of measurement has been chosen which takes for a basis this standard solar negative, as I shall presently show.

In the case of the spectra of Classes II. and III., containing numerous lines, it suggested itself, in determining the displacement between the artificial Hγ and the star line, to connect the former by differential measurements with the other neighbouring lines, not only in order to obtain an increased degree of accuracy, but also indeed from necessity, since, in the generally very small displacements, it wholly or partly concealed the star line. All the difficulties which at first arose in the way of direct measurement of the plates, into the details of which I cannot here enter, were thereby overcome in that I simultaneously with the star spectrum also measured a standard solar spectrum taken with the spectrograph. The plate with the solar spectrum is cut off lengthwise, and so laid upon the star spectrum that the two appear in the microscope one above the other, and separated only by a small space.

The solar negative is observed through the glass, since, in order to avoid parallax, it must be inverted so that the two gelatine films are in contact. It is therefore cut off in order to obviate the effect of the unavoidable impurities in its gelatine film, through which otherwise the star spectrum would have to be observed. It can be then readily brought about that the lines of the one spectrum form the prolongation of those of the other.

The setting upon one of the threads of the system in the eyepiece of the microscope is effected solely by the motion of the above-mentioned sliding apparatus, to which both negatives are firmly attached. In the measurements four settings were usually made on a Sun line, then an equal number on the corresponding star line, those lines being sought out which, generally three on each side, lie nearest Hγ. The Hγ line in the star has only been measured when it was fully separated from the artificial line; but generally, instead of upon it, settings were made six or eight times on the solar Hγ, and an equal number on the artificial Hγ on the star plate.

The difference of the readings on the two spectra gives, in the mean, their displacement as compared with each other as they were brought together under the microscope; this mean, being applied to the difference of the settings on Hγ in the solar negative and the artificial Hγ due to the Geissler tube, gives the actual displacement of the star lines referred to the artificial line.

In most cases the star spectra have been photographed at other temperatures than the Sun, and the differences Sun–star are unequal and show a rate of progression. Under the assump-

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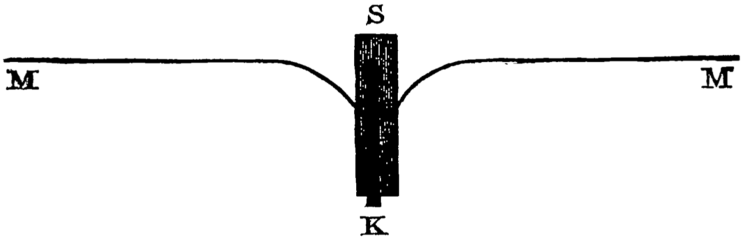
tion of a simple proportional change, which is fully admissible for the small distances which alone come into question, a reduction to the dispersion of the solar negative must first be made, which has been done by the method of least squares.

The great advantage of this method of measurement lies in the fact that the unavoidable distortions in the sensitive film, which, owing to the smallness of the quantities to be measured, might easily come into consideration, are as much as possible eliminated, and also, further, that every prejudice of the observer is entirely eliminated, since the amount of the displacement is not directly obtained, but only after computation.

Most stars of the first spectral class show, in addition to the broad and strongly-marked hydrogen lines, a great number of other lines, so fine, however, that they can be distinctly recognised only in case of the brighter stars. The majority of these lines belong to the iron spectrum, and can be readily identified with solar lines; the measurement of the displacement can therefore either be made by reference to the solar negative, or, under the precautions which I have already stated in the article quoted, the iron spectrum may be used for comparison.

In the case of fainter stars of this class, however, one is restricted to the hydrogen lines, and if the more or less distinctly pronounced maximum of intensity lies outside of the artificial line, then the measurement presents no difficulty; but this is seldom the case, since the breadth of the maximum intensity corresponds to a difference in wave-length of about 0·03 $\mu\mu$, and a displacement of the artificial line only enough to bring its edge into coincidence with the position of maximum intensity would presuppose a velocity of fourteen miles. This maximum of intensity is generally not very sharply bounded, and, with this gradual decrease of the blackening on the negative, it could be accurately observed only in cases of a much greater displacement. The difficulties of this case were therefore insurmountable until a special procedure was thought out, which consisted in covering the Hγ star line, along with the artificial line, by a comparatively broad strip which was brought exactly over the middle of the Hγ line in the star. If, then, the thread of the micrometer be set on the middle of this strip, and the latter be then removed and the thread set on the artificial line, the difference in the readings will give the displacement.



The accompanying figure will make the procedure clearer. M M is the intensity curve of the star spectrum in the vicinity

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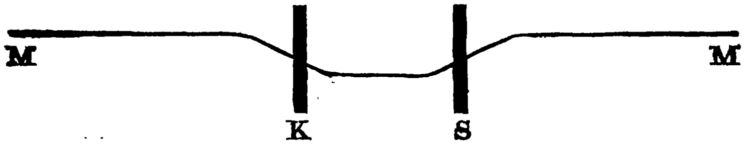
of Hγ, K is the artificial Hγ, displaced from the middle by the star's motion, and S is the strip covering the middle of the position of maximum intensity and the line K.

With a magnification such that the silver grains are readily perceptible, the setting of the strip over the middle of the broad Hγ line can, after some practice, be accurately accomplished by fastening the attention upon the density of the silver precipitate on the plate to right and left of the strip. From a series of small glass plates, upon each of which had been photographically produced a dark strip of differing width (from 0·05 to 0·20 mm.), the suitable one was chosen and laid directly upon the star negative, and fastened by a simple mechanical contrivance which allowed of its being moved backwards and forwards by a fine screw.

When the line in the star spectrum has been very broad and without a distinct position of maximum intensity, I have employed still another method in the measurement of stars with large velocities. A number of lines of varying breadth and blackness, but narrower than the strips just mentioned, were photographed upon small glass plates, and that one was then selected which most nearly equalled in breadth and blackness the artificial Hγ on the negative to be measured. The plate was then laid upon the star negative, and by the fine screw set upon the broad star line, so as to be symmetrical with the artificial Hγ. In the figure the star line is represented by the dipping of the curve M M, K is the line from the Geissler tube, to which is symmetrically set the line S on the glass plate. The micrometric measurement of the distance K S gives the double value of the displacement. By practice much can here be gained in accuracy; attention to the silver grains is in this case also important. It has shown itself to be of advantage to repeat the measurements on different days, since very soon a certain habit of perception sets in which gives the measurements on a single day the appearance of an accuracy which is somewhat illusory.



The first result of any importance which the spectrographic method furnished was the proof of the influence of the Earth's motion on the displacement, which the earlier direct observations had failed to show with certainty. I append here a few examples:—

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Date.	Obs. Vel.	Earth's Vel.	Vel. of Star red. to ☉.
	α Aurigæ.		
1888 Oct. 6	− 3·5	−15·4	+11·9
22	+ 2·9	−13·0	+15·9
24	+ 3·8	−12·6	+16·4
25	+ 3·5	−12·4	+15·9
28	+ 3·8	−11·8	+15·6
Nov. 9	+ 9·2	− 8·9	+18·1
Dec. 1	+10·8	− 2·9	+13·7
13	+15·6	+ 0·7	+14·9
1889 Jan. 2	+20·2	+ 6·6	+13·6
Feb. 5	+30·8	+14·3	+16·5
May 6	+33·2	+17·0	+16·2
Sept. 15	− 3·6	−16·8	+13·2
	α Tauri.		
1888 Oct. 28	+18·1	− 9·5	+27·6
Nov. 10	+24·9	− 5·9	+30·8
Dec. 4	+30·6	+ 1·8	+28·8
1890 Jan. 9	+43·7	+12·3	+31·4
	α Ophiuchi.		
1888 Sept. 30	+27·6	+14·1	+13·5
1889 June 7	+ 9·7	− 1·0	+10·7
	α Ursæ Majoris.		
1888 Nov. 7	−17·0	−11·9	− 5·1
9	−18·1	−11·9	− 6·2
1889 May 4	+ 5·2	+11·8	− 6·6
22	+ 3·4	+11·2	− 7·8

A further result of the new method was the discovery of the changes in the motion of *Algol*, and thereby the proof of the existence of a dark satellite, for the determination of which the most delicate measurements were necessary. The discovery of the periodic motion of *α Virginis* then followed.

As an example of the accuracy of the method of symmetrical setting, which has been applied for stars with broad and ill-defined lines, and generally in the case of *α Virginis*, I give here the results of the measurements, with the period which I have deduced from them :—

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α Virginis. Velocity in the line of sight − velocity of system

$$= 56{\cdot}7 \sin\left(\frac{t-t_0}{p}\right) 360°, \text{ in miles.}$$

Velocity of system = − 9·2 miles. Period, p, = 4·0134 days.

Epoch, t_0, = 1890 May 4, $10^{\text{h}}{\cdot}50$ Potsdam, m.t.

Date.	Obs. Vel. less Vel. of Translation.	Calc. Vel.	O.−C.
1889 April 21	−48·4	−53·5	+5·1
29	−54·3	−52·1	−2·2
May 1	+50·7	+52·6	−1·9
1890 April 4	− 6·5	−12·4	+5·9
9	−55·3	−56·2	+0·9
10	+ 6·0	+10·6	−4·6
11	+50·7	+56·2	−5·5
13	−57·6	−56·2	−1·4
15	+62·7	+56·2	+6·5
May 1	+53·5	+56·7	−3·2
4	− 3·7	− 1·8	−1·9
7	−61·3	−56·7	−4·6
8	− 0·5	− 1·4	+0·9
9	+63·2	+56·7	+6·5
17	+61·8	+56·7	+5·1
18	+ 6·0	+ 5·5	+0·5
23	−65·5	−56·2	−9·3
24	− 1·4	− 5·5	+4·1
25	+53·0	+56·2	−3·2
26	+ 5·1	+ 5·5	−0·4
27	−52·6	−56·2	+3·6
28	−12·9	− 8·8	−4·1
31	−56·2	−56·2	0·0
June 4	−49·8	−55·8	+6·0
1891 April 24	+ 3·2	+ 4·6	−1·4
27	+47·9	+45·2	+2·7
May 3	−58·1	−63·2	+5·1

As an example of the delicacy of the spectrographic negatives, I remark that in those of *β Aurigæ* not only has the periodic doubling of the lines been perceptible and well measurable in the magnesium line $\lambda = 448\ \mu\mu$, but also on some plates in other very fine lines in its vicinity. These observations may be found in *Ast. Nach.*, No. 3017.

In order now to give an illustration of the accuracy which is

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attainable by the method of covering Hγ by a strip, I add my observations on α *Lyræ* and α *Canis Majoris*:—

		Obs. Vel.	Earth's Vel.	Star's Vel. red. to ⊙.
		α Lyræ.		
1888 Sept.	28	− 2·6	+ 8·6	− 11·2
Nov.	11	− 1·7	+ 7·0	− 8·7
	13	− 1·5	+ 6·8	− 8·3
1889 May	31	− 10·7	− 4·8	− 5·9
June	6	− 9·9	− 4·0	− 5·9
Sept.	15	− 1·8	+ 8·1	− 9·9
Nov.	24	− 4·2	+ 5·7	− 9·9
	25	− 6·7	+ 5·6	− 12·3
	26	− 7·2	+ 5·4	− 12·6
		α Canis Majoris.		
1888 Dec.	13	− 14·1	− 4·9	− 9·2
1890 Feb.	12	+ 0·8	+ 9·4	− 8·6
1891 Feb.	7	+ 0·1	+ 8·3	− 8·2
Mar.	21	+ 4·4	+ 13·8	− 9·4
	22	+ 6·1	+ 13·9	− 7·8

I remark, further, that the observations of *Sirius* by the method of stars for the second class give 7·3 miles, and with the aid of the iron comparison spectrum 9·0 miles as the rate of approach towards the Sun.

In regard to the exactness of the measurements in general I will state that out of the average of all the observations the resulting probable error of a single negative for stars of Class II. is ±1·34 mile; for the stars of Class I. ±2·31.

Each star has on the average been observed 3·3 times, and the measurements have been made independently by myself and by Dr. Scheiner. It may, therefore, be concluded that the probable error of the definitive values for both spectral classes will amount to less than one mile.

I intend after the definite completion of the measurements to communicate to the Society a list of the observed velocities, and will remark in conclusion that the velocities of the stars have proved to be much smaller than was to be expected from the direct observations. The mean result for forty-seven stars is 10·6 English miles.

Among them six have a velocity less than 2, and five greater than 20 miles; the greatest is that of α *Tauri*, about 30 miles. Fifteen of the stars have a positive and thirty-two a negative motion.

Potsdam, Royal Observatory:
1891 *December.*

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The Motion of Σ 2525. By S. W. Burnham, M.A.

For some years this has been a difficult pair to measure. When it was discovered and first measured by Struve in 1830, it could be readily seen with the smallest instrument ordinarily used in double-star work. Since that time the distance has gradually lessened, and for the past few years only the largest refractors can separate the components. Until recently the change has been principally in distance, and all the measures made during the first fifty years are well represented by rectilinear motion. Notwithstanding that these positions appear to represent the movement of one star passing another from proper motion, the chances are greatly in favour of a physical relation between them.

I have made a set of measures of this pair with the 36-inch equatoreal under very favourable atmospheric conditions. Although the distance between the components is less than 0″·2, it is easy enough with this instrument, since it is well separated with medium powers, and the measures should have a good degree of accuracy.

The results found on the several nights are ollows:—

	°	″		
1891·326	343·7	0·20	8·0 . . .	8·1
·389	343·4	0·17	8·0 . . .	8·2
·427	163·7	0·17	8·0 . . .	8·5
·449	342·7	0·17	8·0 . . .	8·1
1891·40	343·4	0·18		

In very close pairs of this kind, and particularly when the relative motion is not well understood, it is important to assign the proper quadrant to the smaller star. In the first three measures the angles were set down as above with no thought of the earlier observations, with a note, "quadrant certain," in the second measure. On the following night the angle was reversed, and on the last night the smaller component seemed to be on the preceding side, although the difference in the magnitudes was very small. The magnitudes in Struve are 7·4 and 7·6.

In order to determine this question more certainly, if possible, I examined the pair again on several nights, with the following results:

1891·485. Not very good seeing, but the smaller star appears to be in 160°.

1891·540. The stars are so nearly equal that it is impossible to tell with certainty which is the smaller. Well separated.

1891·578. The preceding star appears to be certainly the smallest, but the difference is perhaps not more than 0·1 or 0·2 of a magnitude. Splendid seeing.